

REVIEW OF CONFIDENCE INTERVALS

- Confidence interval for the mean; σ is unknown

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ is a critical value from T-distribution with $n - 1$ degrees of freedom

- Confidence interval for the difference of means; **equal**, unknown variances

$$\bar{X} - \bar{Y} \pm t_{\alpha/2} s_p \sqrt{\frac{1}{n} + \frac{1}{m}}$$

where s_p is the pooled standard deviation

$$s_p = \sqrt{s_p^2}; \quad s_p^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2 + \sum_{i=1}^m (Y_i - \bar{Y})^2}{n + m - 2} = \frac{(n - 1)s_X^2 + (m - 1)s_Y^2}{n + m - 2}$$

and $t_{\alpha/2}$ is a critical value from T-distribution with $(n + m - 2)$ degrees of freedom.

- Confidence interval for the difference of means; **unequal**, unknown variances

$$\bar{X} - \bar{Y} \pm t_{\alpha/2} \sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$$

where $t_{\alpha/2}$ is a critical value from T-distribution with k degrees of freedom given by the Satterthwaite approximation

$$k = \frac{\left(\frac{s_X^2}{n} + \frac{s_Y^2}{m}\right)^2}{\frac{s_X^4}{n^2(n-1)} + \frac{s_Y^4}{m^2(m-1)}}$$

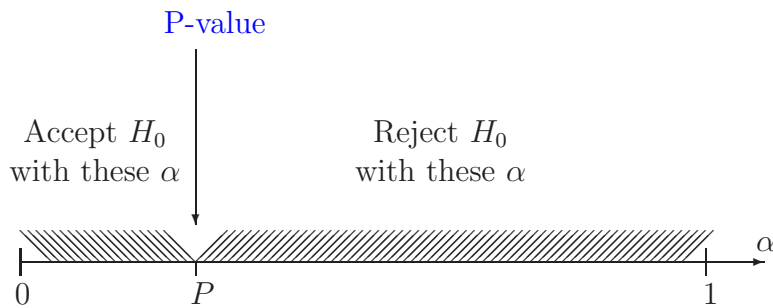
REVIEW OF HYPOTHESIS TESTING

- Summary of T-tests

Hypothesis H_0	Conditions	Test statistic t	Degrees of freedom
$\mu = \mu_0$	Sample size n ; unknown σ	$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$	$n - 1$
$\mu_X - \mu_Y = D$	Sample sizes n, m ; unknown but equal standard deviations, $\sigma_X = \sigma_Y$	$t = \frac{\bar{X} - \bar{Y} - D}{s_p \sqrt{\frac{1}{n} + \frac{1}{m}}}$	$n + m - 2$
$\mu_X - \mu_Y = D$	Sample sizes n, m ; unknown, unequal standard deviations, $\sigma_X \neq \sigma_Y$	$t = \frac{\bar{X} - \bar{Y} - D}{\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}}$	Satterthwaite approximation, see page 1

- **P-value**

P-value is the lowest significance level α that forces rejection of the null hypothesis. P-value is the highest significance level α that forces acceptance of the null hypothesis. P-value is the probability of observing a test statistic that is as extreme as or more extreme than the test statistic computed from a given sample.



- **Computing P-values for T-tests and Stating Conclusions**

Hypothesis H_0	Alternative H_A	P-value	Computation
$\theta = \theta_0$	right-tail $\theta > \theta_0$	$\mathbf{P}\{t \geq t_{\text{obs}}\}$	$1 - F_k(t_{\text{obs}})$
	left-tail $\theta < \theta_0$	$\mathbf{P}\{t \leq t_{\text{obs}}\}$	$F_k(t_{\text{obs}})$
	two-sided $\theta \neq \theta_0$	$\mathbf{P}\{ t \geq t_{\text{obs}} \}$	$2(1 - F_k(t_{\text{obs}}))$

$\left\{ \begin{array}{l} \text{For } \alpha < P, \quad \text{accept } H_0 \\ \text{For } \alpha > P, \quad \text{reject } H_0 \\ \text{Practically,} \\ \text{If } P < 0.01, \quad \text{reject } H_0 \\ \text{If } P > 0.1, \quad \text{accept } H_0 \end{array} \right.$

F_k is the cdf of T-distribution with the suitable number k of degrees of freedom

- **Equivalence of two-sided tests and confidence intervals**

A level α T-test of $H_0 : \theta = \theta_0$ vs $H_A : \theta \neq \theta_0$ accepts H_0 **if and only if** a symmetric $(1 - \alpha)100\%$ confidence T-interval for θ contains θ_0 .

- **Two-sample F-tests for Variances**

Null Hypothesis $H_0 : \sigma_X^2 = \sigma_Y^2$		Test statistic $F_{\text{obs}} = \frac{s_X^2}{s_Y^2}$
Alternative Hypothesis	Rejection region	P-value Use $F(n - 1, m - 1)$ distribution
$\sigma_X^2 > \sigma_Y^2$	$F_{\text{obs}} \geq F_{\alpha}(n - 1, m - 1)$	$\mathbf{P}\{F \geq F_{\text{obs}}\}$
$\sigma_X^2 < \sigma_Y^2$	$F_{\text{obs}} \leq F_{\alpha}(n - 1, m - 1)$	$\mathbf{P}\{F \leq F_{\text{obs}}\}$
$\sigma_X^2 \neq \sigma_Y^2$	$F_{\text{obs}} \geq F_{\alpha/2}(n - 1, m - 1)$ or $F_{\text{obs}} < 1/F_{\alpha/2}(m - 1, n - 1)$	$2 \min(\mathbf{P}\{F \geq F_{\text{obs}}\}, \mathbf{P}\{F \leq F_{\text{obs}}\})$